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LETTER TO THE EDITOR

Dynamic exponent of the 3D Ising spin glass

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Abstract. The non-equilibrium critical dynamics of the 3D Ising spin glass, following a quench to the critical temperature from $T=\infty$, are used to measure both the dynamical exponent z and the static exponent η. The results for z and η are quite sensitive to the value assumed for T_c, although the ratio $(2-\eta)/z$ is rather insensitive, and close to the value 0.39 obtained by Huse. Taking $T_c=1.18$ gives $z=5.85\pm0.3$ and $\eta=-0.29\pm0.07$.

The determination, by conventional Monte Carlo simulation, of the critical exponents at the spin-glass transition has been hampered by the need to fully equilibrate the system before measuring the quantities that determine the exponents. Indeed, this is a problem for any phase transition because of the phenomenon of critical slowing down. For pure systems, this problem can be circumvented to some extent by using one of the algorithms recently developed to accelerate equilibration [1]. Comparable techniques for random systems, however, are still in their infancy [2]. In this letter we introduce two novel (but related) techniques to determine the exponents z and η, quite generally, by exploiting the scaling properties of a system quenched to the critical point from infinite temperature. We illustrate the method by applying it to the 3D Ising spin glass. Taking $T_c = 1.18$, the current best estimate, we obtain $z=5.85\pm0.3$, to be compared with $z=6\pm1$ obtained by conventional methods [3]. The static exponent η is found to be $\eta=-0.29\pm0.07$.

To demonstrate the idea, we first consider a simple Ising ferromagnet quenched from $T=\infty$ to $T=T_c$ at time $t=0$: i.e. we study the equilibration of the system at T_c from an initial state where each spin is randomly up or down. Because relaxation times are infinite at T_c, equilibration is only achieved on a finite length scale $\xi(t)$, a kind of 'non-equilibrium correlation length', which increases with time as $\xi(t)\sim t^{1/z}$. The latter is a consequence of dynamic scaling, which has been shown to hold even for systems out of equilibrium [4].

The first of our two methods is to study the time evolution of the equal time spin–spin correlation function $C(r, t)$, defined by

$$C(r,t)=\langle S(x,t)S(x+r,t)\rangle \tag{1}$$

where $\langle\ldots\rangle$ represents both a thermal average and an average over initial conditions. It is expected to exhibit the scaling form

$$C(r,t)=r^{-(d-2+\eta)}f(r/t^{1/z}) \tag{2}$$

with $f(\infty)=\text{constant}$. The scaling limit is defined by $r\to\infty$, $t\to\infty$ with $r/t^{1/z}$ arbitrary. Equation (2) can be used in principle to determine both η and z from the data. In practice, however, it is very convenient to separately determine the ratio $(2-\eta)/z$ by measuring the 'non-equilibrium susceptibility' $\chi(t) = N[\langle m(t)^2\rangle] = \int d^d r\, C(r, t) \sim t^{(2-\eta)/z}$, where $m(t)$ it the total magnetization per spin at time t and N is the number of spins. With this combination of exponents fixed, one only has to fit a single exponent in (2) to obtain values for both η and z.

In earlier work [5] we have tested this approach on the 2D Ising model. In this case $\eta = \frac{1}{4}$ is known exactly, as is T_c, so z can be fitted directly in equation (2). A very good scaling plot was obtained with $z = 2.15$, consistent with estimates obtained from other methods [6].

For Ising spin glasses, the analogue of equation (1) is

$$C(r, t) = [\langle S(x, t)S(x+r, t)\rangle^2] \tag{3}$$

where $[\ldots]$ indicates an average over the quenched disorder. As before, $\langle\ldots\rangle$ indicates averages over both thermal noise and initial conditions. In order to evaluate $C(r, t)$ within a Monte Carlo simulation we introduce two replicas of the system, i.e. we simulate two independent systems with identical sets of exchange interactions. Then

$$C(r, t) = [\langle S^{(1)}(x, t)S^{(1)}(x+r, t)S^{(2)}(x, t)S^{(2)}(x+r, t)\rangle]. \tag{4}$$

The systems are started from independent initial conditions and subjected to independent thermal noise, i.e. different random numbers are generated for the Monte Carlo updates in each system. The result for $C(r, t)$ is to be compared with the scaling prediction (2). Again the ratio $(2-\eta)/z$ can be determined separately [7] by measuring the 'non-equilibrium spin glass susceptibility' $\chi_{SG}(t) = N[\langle q^2\rangle] = \int d^d r\, C(r, t) \sim t^{(2-\eta)/z}$, where $q = N^{-1}\sum_{i=1}^{N} S_i^{(1)}S_i^{(2)}$.

The second method allows z to be determined completely independently of η. The idea is analogous to the one Binder has proposed [8] to determine the exponent ν, by studying dimensionless ratios of moments of the order parameter to eliminate η. We can readily adapt Binder's approach to non-equilibrium growth by calculating the combination

$$g(t) = \frac{1}{2}\left(3 - \frac{[\langle q^4\rangle]}{[\langle q^2\rangle]^2}\right). \tag{5}$$

At $t = 0$ the spins are random so $q = N^{-1}\sum_{i=1}^{N} S_i^{(1)}S_i^{(2)}$ will have a Gaussian distribution for large N, leading to $g = 0$. In fact, in the thermodynamic limit we expect $g = 0$ for all t because regions whose spatial separation is large compared to $t^{1/z}$ will be statistically independent. Therefore q will have Gaussian distribution for all t: only the width (equal to $(\chi_{SG}(t)/N)^{1/2}$) will change with t. To render equation (5) useful it is necessary to employ finite-size scaling. For a system of linear dimension L one expects $g_L(t) = f(t/L^z)$, with $f(0) = 0$ and $f(\infty) = g_c$ for large L, where g_c is the universal critical value of g in equilibrium. Therefore a plot of $g_L(t)$ versus t/L^z should give a single scaling curve for the correct value of z. With z determined, the finite-size scaling form for the susceptibility, $\chi_{SG} = L^{2-\eta}h(t/L^z)$, can be used to determine η. In order to determine the exponents with precision one requires a reasonable range of values of t/L^z. Achieving the necessary run times restricts us in practice to rather small systems ($L \leqslant 10$).

One major difficulty in the determination of spin glass exponents is the lack of a precise estimate of T_c. From the temperature-dependence of g_L *in equilibrium*, Bhatt and Young [9] estimate $T_c = 1.2^{+0.1}_{-0.2}$. The currently accepted best estimate [3] for the $\pm J$ model is $T_c \simeq 1.18$, and most of the simulations were carried out at this temperature. However, some simulations were also performed at $T = 1.0$, 1.1 and 1.3. In practice it was not possible to pin down T_c precisely from the data. The values of z and η obtained are, unfortunately, quite sensitive to the value assumed for T_c. The value of the ratio $(2-\eta)/z$, however, is somewhat insensitive, and close to the estimate of 0.39 obtained by Huse [7] from the study of $\chi_{SG}(t)$.

In the study of the equal-time correlation function $C(r, t)$, Monte Carlo simulations were performed on 48 pairs of systems, each of size 46^3, with periodic boundary conditions. Data for smaller systems shows that the results are not finite-size affected. Both systems of a given pair have the same exchange interactions, randomly ± 1, but are prepared with independent initial conditions, as discussed above. Runs of 9000 Monte Carlo steps per spin (MCS) were performed using the heat bath algorithm, with parallel updating within a sublattice and sequential updating of the two sublattices, 1 MCS corresponding to a complete update of both sublattices. We emphasize once more that no 'equilibration' is required. Instead we make a virtue of necessity by exploiting the scaling property (2) of the system out of equilibrium.

All the $C(r, t)$ studies were performed at the estimated critical temperature, [3] $T = 1.18$. A *small* error in the location of T_c should not be too important as long as the non-equilibrium correlation length $t^{1/z}$ is much smaller than the equilibrium correlation length ξ at $T = 1.18$. In order to reduce the number of adjustable parameters to one, we exploit a result of Huse [7], who calculated $\chi_{SG}(t) \sim t^{(2-\eta)/z}$ for the 3D $\pm J$ Ising spin glass at the same temperature, $T = 1.18$, and obtained $(2-\eta)/z \simeq 0.39$ by averaging over 2000 realizations of a 10^3 lattice. We checked Huse's result with 500 realizations of a 12^3 lattice and obtained results consistent with (though somewhat noisier than) those of Huse. Our subsequent studies of the function $g_L(t)$, equation (5), confirmed this estimate.

In figure 1 we show $r^{3-0.39z}C(r, t)$ versus $r/t^{1/z}$ for $z = 5.8$, the value which gives the 'best' overall scaling plot. A subjective estimate of when the scaling becomes noticeably worse suggests an uncertainty on z of about 0.4. A number of caveats should, however, be borne in mind. The first point to note is that the length scale $\xi(t) \sim t^{1/z}$

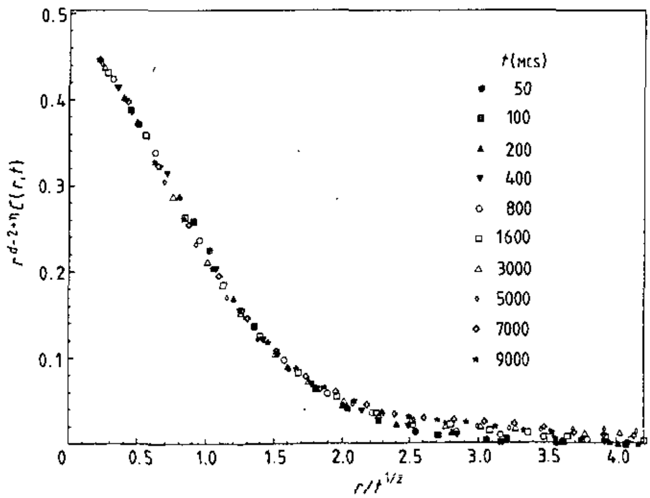

Figure 1. Scaling plot for the equal-time correlation function of the 3D $\pm J$ Ising spin glass quenched from $T = \infty$ to $T = 1.18 \simeq T_c$. We plot $r^{d-2+\eta}C(r, t)$ versus $r/t^{1/z}$ with $(2-\eta)/z = 0.39$ (taken from Huse [7]) and $z = 5.8$. The data represent an average over 48 pairs of 46^3 systems.

is not very large even at the latest times attained, varying from about 2.0 lattice spacings at 50 MCS to about 4.8 lattice spacings at 9000 MCS. On the other hand, the corresponding data for the pure 2D Ising model [5] show no deviations from scaling down to the shortest time (20 MCS) measured, when $t^{1/z} \simeq 4.0$. A second caveat is that the ratio $(2-\eta)/z$ has been taken to be precisely 0.39, the value obtained by Huse [7], who did not quote an error on this number. We have investigated the effect of varying $(2-\eta)/z$ between 0.37 and 0.41, the range of values suggested by our $g(t)$ studies discussed below. Over this range we find no perceptible deterioration in the data collapse if z is adjusted appropriately. The 'best' value of z (obtained from the 'best fit by eye' to the scaling form for $C(r, t)$) is found to vary more or less linearly between 6.15 and 5.40 as $(2-\eta)/z$ is varied from 0.37 to 0.41. Note that our quoted error of 0.4 on z includes both of these extreme values for z. The corresponding 'best estimates' for η vary between -0.28 and -0.21, with a central estimate of -0.26, based on Huse's estimate $(2-\eta)/z=0.39$. Taking this value as given, the estimates of z and η from the $C(r, t)$ data are $z=5.8\pm0.4$ and $\eta=-0.26\pm0.16$. Again, one must bear in mind that these estimates are obtained from rather small values of $\xi(t)$.

The studies of $g_L(t)$ are complicated by the fact that the moments of $P(q)$ are not 'self-averaging' (unlike $C(r, t)$), so that large numbers of systems are needed to get good statistics. In all cases we averaged $g(t)$ over at least 10^3 samples, and in most cases 10^4 samples. The data for $g_L(t)$ are shown in figure 2, where the value of z has been chosen to give the 'best' scaling collapse (judged subjectively) in each case. Where it was impossible to obtain a good collapse of all the data, the $L=4$ data were sacrificed, on the grounds that they are furthest from the scaling regime. We have chosen as abscissa the scaling variable $t^{d/z}/L^d$, rather than simply t/L^z, for the following reason. We note that g_L is basically the fourth cumulant of the distribution $P(q)$, normalized by the square of the second cumulant. To be precise, $g=-\langle q^4\rangle_c/2\langle q^2\rangle^2$, where $\langle q^4\rangle_c \equiv \langle q^4\rangle - 3\langle q^2\rangle^2$ is the fourth cumulant. Provided $\xi(t)$ $(\sim t^{1/z})$, the length scale over which equilibrium has been established at time t, is small compared to the linear dimension

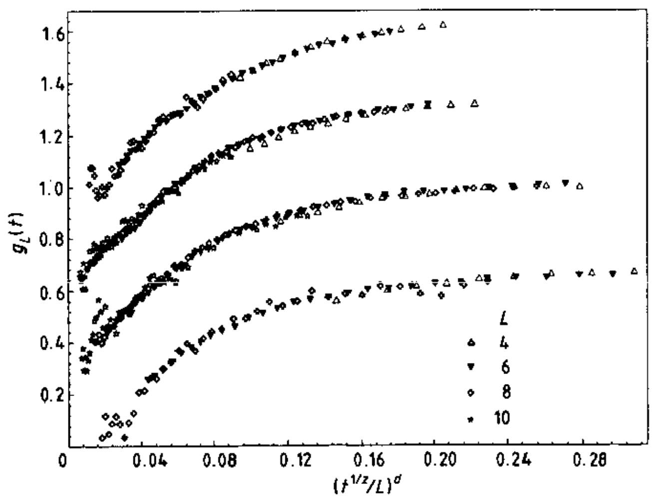

Figure 2. Finite-size scaling plots for $g_L(t)$, equation (5), assuming (top to bottom) $T_c=1.0$, 1.1, 1.18 and 1.3. The corresponding values of z are listed in table 1. To separate the data for different temperatures, additive constants of 0.9, 0.6 and 0.3 have been included for $T=1.0$, 1.1, and 1.18 respectively. Data for 'early times ($t<60$ MCS) have been discarded.

L of the system, $q(t)$ is the sum of $(L/\xi(t))^d$ more or less independent contributions. Under these conditions one finds readily that $g_L(t)$ scales as the reciprocal of this number, i.e. $g_L(t) \sim (t^{1/z}/L)^d$ for $t^{1/z} \ll L$. It follows that the plots in figure 2 should be initially linear, and the data bear this out. In fact a direct plot of ln g versus ln t, for systems somewhat larger than those studied here (to obtain a reasonable dynamic range free of finite-size effects), would have slope d/z, providing a direct measurement of z.

It is clear from figure 2 that the data do not fix T_c with any precision. The values of z used in figure 2 are shown for each temperature in table 1. Again, the errors are subjective, and indicate the limits beyond which the scaling collapse is noticeably poorer than that in figure 2. A value of η for each temperature was deduced from the scaling of the second moment $\langle q^2 \rangle$, using the scaling form $\langle q^2 \rangle = (1/N)\chi_{SG}(t) = L^{2-\eta-d} f(t/L^z)$. Since $\langle q^2 \rangle \sim \xi(t)^{2-\eta}/L^d$ for $t \ll L^z$, we plot $L^{1+\eta}\langle q^2 \rangle$ versus $(t^{1/z}/L)^{2-\eta}$ in figure 3, anticipating once more a linear behaviour initially. This is supported by the data of figure 3, in which η was adjusted at each temperature to give the best scaling collapse, the values of z being those deduced from figure 2. These 'best' values of η are listed in table 1, where the errors are once more subjective and based on the range of η consistent with a reasonable data collapse. The effect of varying z on the

Table 1. Estimates of the exponents z, η and the combination $(2-\eta)/z$ for the 3D Ising spin glass, for various assumed values of T_c, obtained from studies of the function $g_L(t)$. These values are the ones used in the scaling plots of figures 2 and 3.

T_c	1.0	1.1	1.18	1.3
z	6.6 ± 0.4	6.4 ± 0.4	5.9 ± 0.4	5.3 ± 0.6
η	-0.52 ± 0.08	-0.38 ± 0.08	-0.30 ± 0.08	-0.20 ± 0.08
$(2-\eta)/z$	0.38 ± 0.02	0.37 ± 0.02	0.39 ± 0.02	0.415 ± 0.03

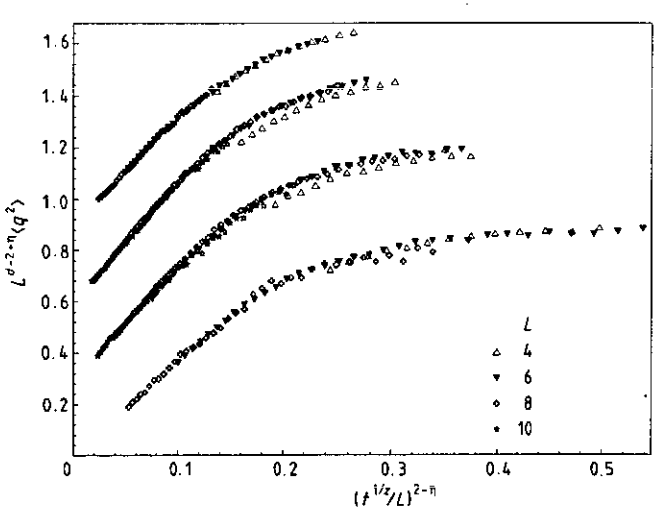

Figure 3. Finite-size scaling plots for $\langle q^2 \rangle$ assuming (top to bottom) $T_c = 1.0$, 1.1, 1.18 and 1.3. The corresponding values of z and η are listed in table 1. To separate the data for different temperatures, additive constants of 0.9, 0.6 and 0.3 have been included for $T = 1.0$, 1.1, and 1.18 respectively. Data for 'early' times ($t < 60$ MCS) have been discarded.

value obtained for η was also investigated. Provided z was moved by no more than the errors given in table 1, the corresponding 'best' value of η was found to be still within the errors for η given in the table.

From table 1 we note that the values of z and η are quite sensitive to the value taken for T_c. However, the variations in z and η are strongly correlated, such that the variation in the ratio $(2-\eta)/z$ is much smaller. The value of this ratio is consistent with that obtained by Huse [7] for $T = 1.18$, namely $(2-\eta)/z \simeq 0.39$.

To summarize, we have introduced a new method (in fact two new methods) of determining the dynamic critical exponent z (and the static exponent η), based on the non-equilibrium growth of critical correlations in a system quenched to T_c from $T = \infty$. These methods have the advantage over conventional methods that no Monte Carlo time is wasted equilibrating the system. The methods have been used to obtain an estimate of z for the 3D Ising spin glass. By measuring the equal time correlation function (1), taking $(2-\eta)/z = 0.39$ from previous work of Huse [7], and assuming $T_c \simeq 1.18$, we obtain $z = 5.8 \pm 0.4$ where the errors are estimated subjectively from the quality of the data collapse. The corresponding estimate for η is $\eta = -0.26 \pm 0.16$. This is consistent with the estimate $\eta = -0.3 \pm 0.2$ obtained by Bhatt and Young [9]. Measurements of the function $g(t)$ defined by (5), combined with finite-size scaling analyses, give, for an assumed critical temperature $T_c = 1.18$, the estimates $z = 5.9 \pm 0.4$ and $\eta = -0.30 \pm 0.08$. Combining the estimates from the two different methods gives the final results $z = 5.85 \pm 0.03$ and $\eta = -0.29 \pm 0.07$. Previous estimates of z and η, based on $T_c = 1.175$ and 1.2 respectively, are $z = 6 \pm 1$ [3] and $\eta = -0.3 \pm 0.2$ [9], so the current method provides a more precise determination of both exponents than was previously available.

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